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Anomalous scale dimensions from time-like braiding

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Abstract

Using previously gained insight about the particle/field relation in conformal quantum field theories, which required interactions to be related to the existence of particle-like states associated with fields of anomalous scaling dimensions, we set out to construct a classification theory for the spectra of anomalous dimensions. Starting from the old observations on conformal superselection sectors related to the anomalous dimensions via the phases which appear in the spectral decomposition of the centre of the conformal covering group $Z(\widetilde{SO(d,2)})$, we explore the possibility of a time-like braiding structure consistent with the time-like ordering which refines and explains the central decomposition. We regard this as a preparatory step in a new construction attempt of interacting conformal quantum field theories in $d = 4$ spacetime dimensions. Other ideas of constructions based on the AdS_5–CQFT_4 or the perturbative SYM approach in their relation to the present idea are briefly mentioned.

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1. Background and preview of new results

It has been known for a long time that conformal quantum field theory exhibits, in addition to the general spin-statistics theorem, another more characteristic structural property which we refer to as the 'anomalous dimension–central phase' connection. This relates the anomalous scale dimension of fields modulo integers (semi-integers in the case of fermion fields) to the phase obtained by performing one complete time-like sweep around the compactified Minkowski world [1] and hence is analogous to the univalence superselection rule of the semi-integer spin which historically spells the beginning of the issue of superselection rules in the well known paper of Wick *et al* [36]. In the spin case, one associates with a 2π spatial rotation sweep the statistics phase $(-1)^{2s}$ of the spin-statistics connection [2] and the question of whether there is a commutation relation behind the superselected coherent subspace corresponding to each

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phase factor of the time-like sweep poses itself naturally. The word 'central' here refers to the centre $Z(\widetilde{SO(d,2)})$ of the infinite-sheeted covering group $\widetilde{SO(d,2)}$ which has one Abelian generator Z for spacetime dimensions $d > 1+1$. In chiral conformal theories the sweep is light-like and the spin and its time-like analogue coalesce, whereas in higher dimensions there is the problem of consistency with the well known DHR superselection theories of internal symmetries based on space-like commutativity.

It is our aim to show that the analogy is deeper than expected at first sight: namely, the statistics aspect of the spin has an algebraic counterpart in the form of a time-like braid group ('plektonic') commutation relation. Here the notion of global causality in the covering of the compactified Minkowski spacetime is important because it was on the basis of this concept that the 'Einstein causality paradox' [8] was solved [1]. The conformal decomposition theory resulted from the attempt to avoid the covering formalism (which is not natural from a particle physics viewpoint) and to deal instead with projected fields which behave like sections over compactified Minkowski spacetime rather than globally causal fields on the covering. Whereas the latter are Wightman fields, the former are not since they carry with them a source and a range projector[2] and hence are similar to the exchange algebra fields of chiral theories [25]. As in the chiral case, the spectrum of anomalous dimensions (possibly modulo a common Abelian contribution) is determined in terms of the admissible braid group representations. The chiral observables on S^1 correspond to conformal observables on compactified Minkowski space $\bar{M}$. For the latter one has space- and time-like commutativity (validity of Huygens principle). In fact, this distinction and the notion of causality altogether becomes meaningless and only light-like distances have an invariant meaning.

In $d = 1+1$ dimensions one has accumulated a good understanding of conformal theories and in particular of their associated superselected charge structure. One knows that they can be decomposed into the $x_\pm$ chiral light cone components. There is a systematic way to classify localizable representation of chiral observable algebras (at least in principle) and one finds charge-carrying fields which obey a light-like exchange algebra [25, 26] in which those new objects satisfy braid group commutation relations either of the Abelian kind (anyonic) or with plektonic (which includes the non-Abelian case) R matrices with quantized statistical phases. Since the latter determine the spectrum of anomalous dimensions (or spectrum of 'twists'= scale dimension minus spin) modulo integers, one has a theory of anomalous dimension as soon as one knows how to classify physically admissible representations of the infinite braid group or, more precisely, the ribbon braid group RB_∞. The classification of the latter is done by the method of tracial states on B_∞ which follow a combinatorial version of the field-theoretic cluster decomposition property, the so-called Markov property [5, 34]. This method was originally devised in the early 1970s by Doplicher, Haag and Roberts (DHR) in order to classify the admissible permutation group statistics which is associated with the algebraic superselection theory of compactly localized charges in $d > 1+1$ [3] in a formulation without field multiplets. In subfactor theory [4] this method was independently devised in a far more general context and called, very appropriately, the method of 'Markov traces' (due to Vaughn Jones) which in turn was gratefully re-adopted by the physicists. The name Markov in this context reveals a lot about the conceptual scope of this theory because while it refers to Markov junior, the Russian mathematician who made an important contribution to the early study of the braid group, at the same time one is invited to think of Markov senior the probabilist since, while for a physicist the property of this tracial state (which then allows its iterative determination) is a discrete version of the field-theoretic cluster decomposition property, to a

[2] Such operators are sometimes called 'vertex operators'. The old terms 'central component or projected fields' [1] or the more recent name 'exchange algebra fields' better suit the content of the present paper.

mathematician this procedure is more reminiscent of a discrete stochastic process. A field-theoretic version of the classification of admissible braid group representations based on the Markov trace formalism can be found in [6, 13, 14, 35]

The reader will notice that in our enumeration of achievements in chiral theories we have omitted the better-known representation theory of specific algebras as the energy–momentum tensor algebra or current algebras. This is not to ignore their important role in the modern development of chiral theories but is, rather, a result of the fact that they have no direct counterpart in higher dimensions. So if we want to use chiral theory as a theoretical laboratory for higher-dimensional conformal field theories we are forced to de-emphasize those aspects and highlight instead others, such as space- and time-like commutation structures which are independent of spacetime dimensions.

The weak point of the present approach to higher-dimensional conformal theories is, of course, the total lack of nontrivial examples, i.e. of higher-dimensional conformal models with anomalous scaling dimensions. As already mentioned, the new time-like superselection structure can unfortunately not be illustrated by the representation theory of any known algebra (unlike chiral theory) and it is also not possible to explore this time-like region by Lagrangian methods (which tend to favour the Euclidean or space-like regions). There is, of course, the folklore that the only Lagrangian conformal four-dimensional theories are a special kind of supersymmetric Yang–Mills theory which, if it could be made more rigorous in a clearer conceptional setting (less computational recipes, more physical principles), would be a remarkable observation. The present non-Lagrangian approach suggests another picture: instead of a scarceness of models one should expect a similar wealth as in the case of nonperturbative chiral theories. I believe that in the near future we will have new concepts and methods for their construction from time-like braid group data (see concluding remarks, section 4).

Given the lack of illustrative examples which could show that the present requirements allow for nontrivial realizations, we are limited to consistency checks. The main new problem which was absent in the chiral case is the consistency of time-like braiding with space-like locality: i.e., one has to show that the new time-like plektonic structure is in harmony with the standard boson/fermion local commutation relations for the space-like region. This will be the subject of section 3.

The next section contains a review of the geometric setting of conformal symmetry. Although most of these results have been known since the 1970s, we find it convenient for setting notation and concepts to present an updated version. The isomorphism of conformal field theory with QFT in 5D anti-de Sitter (AdS) spacetime belongs logically (though not historically) to that section which deals with issues of compactification, covering and global causality.

Some comments on the relation of conformal QFT and particle physics are in order. In most of the recent literature, standard notions of particle physics and scattering theory have been used in the conformal setting without qualifications. But a closer examination shows that conformal field theory is not a theory of interacting particles, at least not if these concepts are still used in a way which is not completely void of their original physical meaning. There are no discrete zero-mass shells (light cone delta functions in momentum space) in an *interacting* dilation-invariant theory and there is *a fortiori* no LSZ S matrix (the asymptotic LSZ limits in fact vanish). Any kind of interaction dissolves immediately the zero-mass shell into the continuum with a continuous mass distribution of enhanced weight at $p^2 = 0$. There remains, of course, the interesting question of what kind of residual particle physics information one can extract from the scale-invariant limit of a massive particle theory[3]. For a discussion of this and related points see [15]. The structural problems of conformal QFT to which we draw attention in what follows do not depend on particle interpretations.

2. Covering space and decomposition theory

Since massless particles in a conformal theory cannot interact [15], the physically interesting interacting fields in a conformal theory are those with anomalous dimension. Whereas conformal free fields are commutative in the Huygens sense of time- and space-like commutativity (Huygens principle in wave optics transposed into the setting of local quantum physics), conformal interactions lead to 'reverberations' inside the light cone which are in correspondence with the appearance of anomalous dimension and produce a paradoxical violation [8] of Einstein causality if one uses the standard description of Minkowski space and allows 'big' special conformal transformations to act on space-like vanishing commutators with the usual transformation rules. Hence, it was of interest to investigate the global spacetime interpretation of such fields in more detail in order to remove the paradox. For aspects of global localization in interacting conformal field theories one needs to introduce the covering of the conformally compactified Minkowski space with its nontrivial topology. This is a well studied old subject [1,21,22], which led to an important global decomposition theory which, after lying dormant for a number of years (when gauge theories took the centre of the stage) [9], obtained an unexpected fresh impetus in the special case of $d = 1+1$ [16] where conformal observables decompose (similar to free fields) additively into the two light cone components. The latter act on a tensor-factorized Hilbert space whereas charge-carrying fields act on a tensor product of extended superselected Hilbert spaces.

The fastest way to obtain a first glance at the formalism and physical use of the conformal covering space is to note that the Wigner representation theory for the Poincaré group for zero-mass particles allows an extension to the conformal symmetry (without extending the Hilbert space and the degree of freedoms): Poincaré group $\mathcal{P}(d) \to SO(d,2)$. Besides scale transformations, this larger symmetry also incorporates the fractional transformations (proper conformal transformations)

$$\begin{aligned} x' &= \frac{x - bx^2}{1 - 2bx + b^2x^2} = IT(b)I \\ I : x &\to \frac{-x}{x^2} \qquad T(b) : x \to x + b. \end{aligned} \tag{1}$$

The conformal reflection I itself is not a Möbius transformation, but in free field theories it is known to be implemented by a unitary transformation [9]. For fixed x and small b the formula (1) is well defined, but globally it mixes finite spacetime points with infinity and hence requires a more precise definition (in particular, in view of the positivity energy–momentum spectral properties) in its action on quantum fields.

As a preparatory step for the QFT concepts one has to achieve a geometric compactification. This starts most conveniently from a linear representation of the conformal group $SO(d,2)$ in six-dimensional auxiliary space $\mathbb{R}^{(d,2)}$ (i.e. without field-theoretic significance) with two time-like signatures

$$G = \begin{pmatrix} g_{\mu\nu} & & \\ & -1 & \\ & & +1 \end{pmatrix} \qquad g = \mathrm{diag}(1,-1,-1,-1) \tag{2}$$

[3] Although scattering amplitudes diverge or go to zero in the scaling limit, highly inclusive cross sections could stay finite.

and restricts this representation to the $(d+1)$-dimensional forward light cone

$$LC^{(d,2)} = \{\xi = (\boldsymbol{\xi}, \xi_4, \xi_5\} \boldsymbol{\xi}^2 - \xi_d^2 + \xi_{d+1}^2 = 0\} \tag{3}$$

where $\boldsymbol{\xi}^2 = \xi_0^2 - \vec{\xi}^2$ denotes the d-dimensional Minkowski length square. The compactified Minkowski space is obtained by adopting a projective point of view (stereographic projection)

$$M_c^{(d-1,1)} = \left\{ x = \frac{\boldsymbol{\xi}}{\xi_d + \xi_{d+1}}; \xi \in LC^{(d,2)} \right\}. \tag{4}$$

It is then easy to verify that the linear transformation which keeps the last two components invariant consists of the Lorentz group, and those transformations which only transform the last two coordinates yield the scaling formula

$$\xi_d \pm \xi_{d+1} \to \mathrm{e}^{\pm s}(\xi_d \pm \xi_{d+1}) \tag{5}$$

leading to $x \to \lambda x, \lambda = \mathrm{e}^s$. The remaining transformations, namely the translations and the fractional proper conformal transformations, are obtained by composing rotations in the $\boldsymbol{\xi}_i - \xi_d$ and boosts in the $\boldsymbol{\xi}_i - \xi_{d+1}$ planes.

The so-obtained spacetime is most suitably parametrized in terms of a 'conformal time' τ:

$$\begin{aligned} &M_c^{(d-1,1)} = (\sin\tau, e, \cos\tau) \qquad e \in S^{d-1} \\ &t = \frac{\sin\tau}{e^d + \cos\tau} \qquad \vec{x} = \frac{\vec{e}}{e^d + \cos\tau} \\ &e^d + \cos\tau > 0 \qquad -\pi < \tau < +\pi \end{aligned} \tag{6}$$

so that the conformally compactified Minkowski space is a piece of a multi-dimensional cylinder carved out between two $(d-1)$-dimensional boundaries which lie symmetrically around $\tau = 0, e = (\mathbf{0}, e^d = -1)$ where they touch each other [22] (figure 1 below). If one cuts the cylinder wall this region $\bar{M}^{(d-1,d)}$ looks like a double cone subtended by two points at infinity $m_+(\tau = \pi, \vec{e} = 0, e^d = 1), m_-(\tau = -\pi, \vec{e} = 0, e^d = 1)$; the boundary region $\bar{M}\backslash M$ consists precisely of all points which are past/future light-like from m_+/m_- (the light-rays on the cylinder are continuous lines starting from m_- and ending at m_+ which are points to be identified on $\bar{M}$). In this way the cylinder is equipped with a tiling into infinitely many ordinary Minkowski spaces:

$$\widetilde{M_c^{(d-1,1)}} = S^{d-1} \times \mathbb{R}. \tag{7}$$

Whereas matter in $\bar{M}$ is subject to the quantum version of the Huygens principle (namely, observables commute if the light ray subtended by one localization region cannot reach the other), the covering $\tilde{M}$ comes with a global causality structure. The relevance of this covering space for the notion of relativistic causality was first pointed out by Segal [21] and the above parametrization as well as many other contributions which became standard in conformal QFT appeared for the first time in the work of Luescher and Mack [22].

Formally, this framework solves the 'Einstein causality paradox of conformal quantum field theory' [8] which originated from 'would-be' conformal models (locally conformal invariant) of QFT such as, for example, the massless Thirring model which violates the Huygens principle. The naive reason for this apparent violation turned out to be that there exist continuous curves of special conformal transformations which lead from time-like separations with one point at the origin via the light-like infinity to space-like separation. This obviously generates a contradiction with the locality structure of the Thirring model whose time-like anti-commutator, unlike the space-like one, does not vanish. The covering structure formally solves this causality paradox by emphasizing that in fact there are no covering transformations which violate global causality; a paradox only arises if points become projected out of $\tilde{M}$

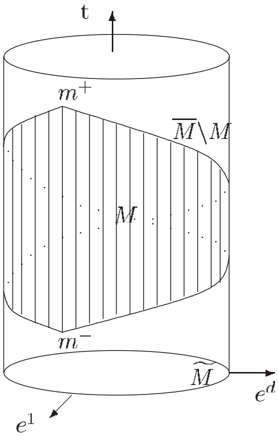

Figure 1. An embedding of the Minkowski space into the manifold M.

into M. In other words, one must keep track of the global path which remains space/time-like and not just its end points. If one depicts, as before, the covering space as a cylinder (figure 1), then it contains infinitely many copies of the original Minkowski space which appear in the projection to $(\xi_2, \ldots, \xi_{d-1}) = (0, \ldots, 0)$ subspace as a finite rhomboid region [22].

Using the above parametrization in terms of e and the 'conformal time' τ, one can immediately globalize the notion of time-like distance and one finds the following causality structure [21, 22]:

$$\begin{aligned} &\left(\xi(e,\tau) - \xi(e',\tau')\right)^2 \lessgtr 0 \qquad \text{hence} \\ &\left|\tau - \tau'\right| \lessgtr \left| 2\arcsin\left(\frac{e-e'}{4}\right)^{\frac{1}{2}} \right| = \left|\arccos\left(e \cdot e'\right)\right| \end{aligned} \tag{8}$$

where $\lessgtr$ now denotes the space-like/($\pm$)time-like separation in the global sense. Since it is expressed in terms of the difference of two coordinates on the light cone in the $(d+2)$-dimensional ambient spacetime, a conformal transformation which is linear in the ξ-variables leaves it invariant. For the description of the Dirac–Weyl compactified Minkowski space, the use of the following simpler parametrization close to standard Minkowski coordinates is more convenient:

$$\begin{aligned} &\xi^\mu = x^\mu \\ &\xi^4 = \tfrac{1}{2}(1 + x^2) \\ &\xi^5 = \tfrac{1}{2}(1 - x^2) \\ &\text{i.e.} \;\; \left(\xi - \xi'\right)^2 = -\left(x - x'\right)^2 . \end{aligned} \tag{9}$$

Similarly, one may use the quadratic polynomial $\sigma(b, x)$ appearing in the denominator (and the Jacobian) of the special conformal transformations in order to decide whether two points are globally time-like/space-like (connectable by time-like/space-like geodesics) without using (8) [1, 3].

The formulation in terms of conformal covering space would be useful if the world (including laboratories of experimentalists) were also conformal, which certainly is not the case. Therefore, it is helpful to know that there is a way of rephrasing the physical content of local fields (which violate the Huygens principle and instead exhibit the phenomenon of 'reverberation' [8] inside the forward light cone) in the Minkowski world M of ordinary particle physics without running into the trap of the causality paradox of the previous section; in this way the use of the above ξ-parametrization would lose some of its importance and this may be considered as an alternative to the Lüscher–Mack approach on covering space.

Such an approach was first followed in a joint paper involving the present author [1]. The main point was that the global causality structure could be encoded into a global decomposition theory of fields with respect to the centre of the conformal covering (conformal block decomposition). Local fields, although behaving apparently irreducibly under infinitesimal conformal transformations, transform, in general, reducibly under the action of the global centre of the covering $Z(\widetilde{SO(d,2)})$. This reduction was precisely the motivation for the global decomposition theory of conformal fields [1]:

$$F(x) = \sum_{\alpha,\beta} F_{\alpha,\beta}(x), \quad F_{\alpha,\beta}(x) \equiv P_\alpha F(x) P_\beta$$
$$Z = \sum_{\alpha} \mathrm{e}^{2\pi \mathrm{i}\theta_\alpha} P_\alpha. \tag{10}$$

These component fields behave in an analogous way to trivializing sections in a fibre bundle; the only memory of their origin from an operator on covering space is their quasiperiodicity:

$$Z F_{\alpha,\beta}(x) Z^* = \mathrm{e}^{2\pi \mathrm{i}(\theta_\alpha - \theta_\beta)} F_{\alpha,\beta}(x) \tag{11}$$

$$U(b) F_\delta(x)_{\alpha,\beta} U^{-1}(b) = \frac{1}{[\sigma_+(b,x)]^{\delta-\zeta}\left[\sigma_-(b,x)\right]^{\zeta}} F_\delta(x)_{\alpha,\beta}$$
$$\zeta = \tfrac{1}{2}(\delta + \theta_\beta - \theta_\alpha) \tag{12}$$

where the second line is the transformation law of special conformal transformation of the components of an operator F with scale dimension δ sandwiched between superselected subspaces H_α and H_β. Using the explicit form of the conformal 3-point function it is easy to see that phases are uniquely given in terms of the scaling dimensions δ which occur in the conformal model [1]:

$$\mathrm{e}^{2\pi \mathrm{i}\theta} \in \begin{cases} \{\mathrm{e}^{2\pi \mathrm{i}\delta} \mid \delta \in \text{scaling spectrum}\} & \text{bosons} \\ \{\mathrm{e}^{2\pi \mathrm{i}(\delta+\frac{1}{2})} \mid \delta \in \text{scaling spectrum}\} & \text{fermions.} \end{cases} \tag{13}$$

A central projector projects onto the subspace of all vectors which have the same scaling phase i.e. onto a conformal block associated with the centre, so the labelling refers to (in the case of bosons) the anomalous dimensions mod 1. These subspaces of operators are bigger than the chiral blocks, since the anomalous dimension does not, for example, distinguish between with charged fields and their anti-fields which carry the conjugate charge. Having an understanding of the physical interpretation of the central decomposition does not mean that we yet have a theory of the spectrum of admissible anomalous dimensions (critical indices) in higher-dimensional conformal theories, but the close analogy to chiral theories gives sufficient incentive to look for such a theory. This is the subject of the next section.

The price one has to pay for this return to the realm of particle physics on M in terms of component fields (10) is that these projected fields are not Wightman fields. They depend on a source and range projector, and if applied to a vector the source projector has to match the Hilbert space: i.e., $F_{\alpha,\beta}$ annihilates the vacuum if P_β is not the projector onto the vacuum sector. This is very different from the behaviour of the original F which, in the case where it is localized in a region with a nontrivial space-like complement, can never annihilate the vacuum. This kind of projected field is well known from the exchange algebra formalism of chiral QFT [25]. They also appeared in a rudimentary form in [1].

Inside 2- and 3-point functions the projectors are unique and may be omitted, but for $n \geqslant 4$ there are several projected n-point functions and therefore they are needed.

Structural properties of the real-time formulation such as this time-like decomposition formula remain totally hidden in the Euclidean formulation. They lead to cuts with multi-valuedness in an analyticity region which is beyond the standard BHW [2] extended permuted tube region (see next section) of standard Poincaré-invariant theories. As in the chiral case, one defines conformal observable fields as those which commute with the centre generator Z and, as a consequence, are free of these cuts. That is to say, they have meromorphic (rational) correlation functions on $\bar{M}$ and its complex extensions, as is well known from the chiral conformal observables, where the terminology 'holomorphic' was unfortunately attributed to the observable fields[4] instead of referring to a particular state.

The structure of the centre in chiral conformal field theories is determined by the discrete spectrum of the rotation operators for the compactified $\pm$ light-rays $R^{(\pm)} = L_0^{(\pm)}$, where the right-hand side is the standard Virasoro algebra notation. It is well known that this operator shares with the light-ray translations $P^{(\pm)}$ the positivity of its spectrum. This becomes obvious if one represents it in terms of P:

$$\begin{aligned} R^{(\pm)} &= P^{(\pm)} + K^{(\pm)} \\ K^{(\pm)} &= I^{(\pm)} P^{(\pm)} I^{(\pm)} \end{aligned} \tag{14}$$

where $I_{\pm}$ represents the chiral conformal reflection $x \to -\frac{1}{x}$ (in linear light-ray coordinates x) and K is the generator of the fractional special conformal transformation (1). However, the 2D inversion does not factorize since the chiral inversion rewritten in terms of 2D vector notation corresponds to

$$\begin{aligned} x_0 &\to -\frac{x_0}{x^2} \\ x_1 &\to \frac{x_1}{x^2}. \end{aligned} \tag{15}$$

The 'wrong' sign in the spatial part can be corrected by a parity transformation $x_+ \leftrightarrow x_-$ which mixes the two chiral components. In defining an object which transforms as a vector this has to be taken in consideration:

$$R_\mu = P_\mu + I P_\mu I \tag{16}$$

$$I = \text{parity} \cdot \text{ inversion}. \tag{17}$$

The vector formula (16) is valid in any dimension: that is, it does not require light-ray factorization. It leads to a family of operators with discrete spectrum $e \cdot R$ which are dependent on a time-like vector e_μ. As in the chiral case, one only needs to add to the Poincaré + scale transformations the (time-like) conformal rotation R_0; the other components of R_μ are generated by the action of the Lorentz group.

To understand the geometric action of $\mathrm{e}^{\mathrm{i}e \cdot R\tau}$, it is helpful to depict the covering world $\tilde{M}$ with a copy of the Minkowski world inside. From (6) one obtains the identification of the covering world with the surface of a $(d+1)$-dimensional cylinder [22]. In figure 1 only two of the $d-2$ components of the d-dimensional $\boldsymbol{e}$-vector have been drawn; the others have been set to zero. For depicting the space-like complement of a double cone $\mathcal{O}$ in $\tilde{M}$ it is more convenient to cut open the cylinder in the τ-direction and identify opposite sides as in figure 2.

On the other hand, the living space of the observable algebra is the Dirac–Weyl compactification $\bar{M}$ of M, which is depicted in figure 3 with two opposite sides a and b identified. Vice versa, the Minkowski space results from puncturing the Dirac–Weyl compactification $\bar{M}$ at $m_+ = m_-$ and simultaneously removing the whole subtended light-like $(d-1)$-dimensional subspace. Note that, as a result of the identification, the union of the time-like and space-like complements form a connected set in $\bar{M}$; in fact, that part of the space-like complement of a double cone $\mathcal{O}$ in the covering $\tilde{M}$ beyond M becomes converted into the

[4] There are no holomorphic local fields in QFT, not even for free fields. The correlation functions have analytic properties which depend sensitively on the state in which the correlations are studied: for example, they are very different in ground states than in thermal states, although the local operator algebras remain the same.

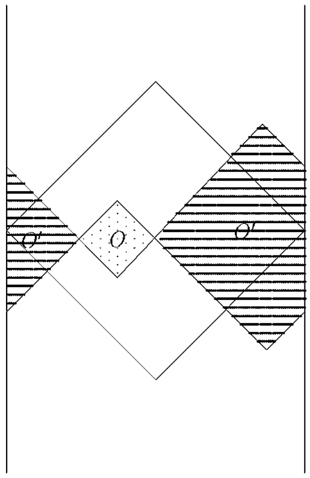

Figure 2. The space-like complement of a double cone O in M within $\tilde{M}$.

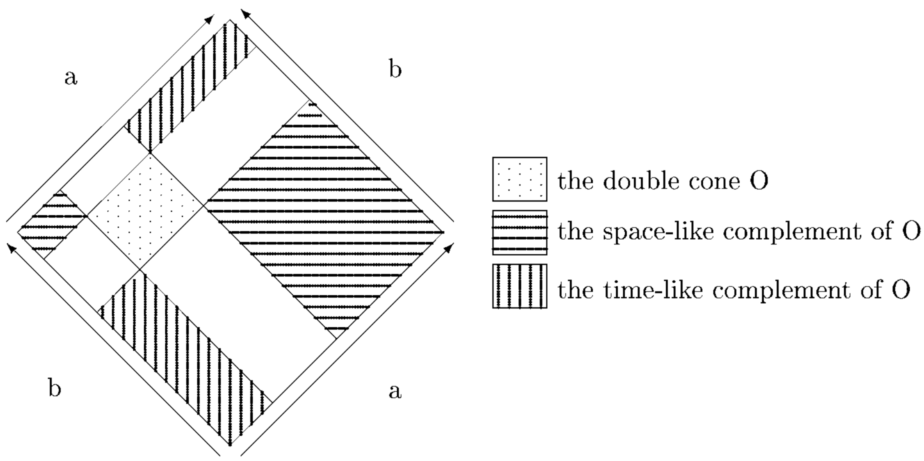

Figure 3. The Dirac–Weyl compactification.

Huygens time-like region with respect to $\mathcal{O}$. The first use of these geometrical properties in the setting of algebraic QFT is due to Hislop and Longo [23]. The pictures are closely related to those used by Penrose, except that Penrose does not use them for compactification since he is dealing with a conformal class of spacetime metrics and not with conformal-invariant observable matter fulfilling the Huygens principle.

It is hard to resist mentioning that the $(d+2)$-dimensional setting for the compactification and subsequent covering of d-dimensional Minkowski spacetime also lends itself to obtaining a natural relation with the $(d+1)$-dimensional AdS spacetime by taking, instead of the surface of the light cone, a hyperbolic region inside this light cone $\xi^2 = 1$. It is immediately clear that, asymptotically, this $(d+1)$-dimensional Lorentzian noncompact manifold in the associated ξ-parametrization has a d-dimensional conformal boundary which in the above picture corresponds to the asymptotic coalescence of the AdS_{d+1} with the light cone directions. This asymptotic point-like relation can, of course, not be continued inside the AdS_{d+1} spacetime, but the action of the same $SO(d, 2)$ group on the two manifolds suggests a relation between d-dimensional conformal double cone (conformal transforms of Rindler wedge regions) and $(d + 1)$-dimensional wedge regions on AdS_{d+1}: i.e. between those regions which result from projecting $(d+2)$-dimensional wedges in the ambient space (on which there is a transitive linear action of $SO(d, 2)$) onto $\bar{M}_d$ and AdS_{d+1}. Using the setting of algebraic QFT, Rehren [18] converted this geometric relation into an isomorphism between algebraic QFTs where the principal objects are localized algebras and not their coordinatizations in terms of point-like field generators. In fact, as one naively expects, the isomorphism negates a relation between point-like field theories (see below).

Although from a logical point of view the above observation on AdS spacetime belongs naturally to the conformal compactification setting of the 1970s, when most of the above observations were made, history (as almost always and in particular in this case) did not follow logic. Rather, part of the isomorphism, namely the mapping $AdS_{d+1} \overset{\text{asympt.}}{\rightarrow} \bar{M}_d$ for the corresponding QFTs, was first observed by string theorists at the end of the 1990s [17] in connection with their speculative ideas on quantum gravity. Although the underlying idea was that the asymptotic map characterizes a unique AdS theory once the conformal asymptote has been prescribed, it was Rehren's construction which supplied a constructive mathematical proof for the full isomorphism and at the same time highlighted the conceptual scope of (field-coordinate-free) local quantum physics.

The isomorphism can be brought closer to the realm of particle physics if one highlights the above analogy between the Hamiltonian H and the conformal 'Hamiltonian' R_0 by asking the following question: is there a QFT which maintains the symmetries but for which the conformal Hamiltonian of the M_d becomes the true Hamiltonian? The answer is unique, it is precisely the same AdS_{d+1} theory of the Rehren isomorphism which changes the physical interpretation and the spacetime affiliation, but not the group-theoretical and algebraic net structure.

The particle physics nature of this isomorphism can be further clarified by studying concrete examples: for example, the fate of free conformal/AdS fields under this isomorphism. The result is rather interesting [37]. A point-like AdS free field has too many degrees of freedom with the result that its conformal image is a special 'generalized free field' with a homogeneous mass distribution which destroys the primitive causality, i.e. the requirement that the algebraic data in a time slice (in order to avoid short-distance problems of space-like surfaces) which covers a compact spatial region fix the data in the 'causal shadow' region (the double-cone-shaped causal envelope). Using Rehren's graphical illustration [19], which depicts conformal QFT on a cylindrical boundary of the AdS world, one sees that there are more and more degrees of freedom from the inside of AdS entering the causal shadow as one moves upward in time. Vice versa, if one starts from a free conformal theory then the AdS image cannot sustain point-like fields but only configurations which are delocalized (and hence diluted) in one direction, such as some kind of Nielsen–Olsen string: i.e. the memory from its d-dimensional point-like origin is stored in the AdS image. This means, in practical terms, that the nice idea of starting with a Lagrangian AdS theory and reprocessing with the Witten prescription to the conformal side in order to enrich the set of conformal models does not work. One obtains conformal theories in this way but they are unphysical, a fact which remains concealed in the Euclidean formulation.

This also affects the conjecture's string theory–SYM relation to the extent that it relies on a relation between AdS_5 and conformal SYM Lagrangian theories.

The algebraic isomorphism itself is not limited by these deficiencies in point-like (Lagrangian) relations and there remains the intellectually challenging problem of understanding the algebraic conformal decomposition theory and its DHR origin directly for QFT on AdS. Just because the isomorphism changes the interpretation, it is by no means obvious, without looking carefully at the details, what the observations in this paper mean in an AdS spacetime. Since the spectrum of anomalous dimensions becomes encoded into the spectrum of a Hamiltonian, one could be optimistic and hope for certain simplifications on the AdS side. A rather trivial, and unfortunately atypical, case is AdS_2 whose bona fide Hamiltonian is the Virasoro L_0—in fact it is the only system with a maximal $SL(2, R)$ symmetry having that compact (discrete spectrum) operator as its Hamiltonian.

The above analogies between the $d = 1 + 1$ case and the higher-dimensional conformal field theory should, however, not lead one into overlooking a remarkable difference. Already on a purely classical level the characteristic value problem for the free-wave equation is totally different from either its massive counterpart or from the $d > 2$ case. Whereas in the latter cases the data on one light-ray or lightfront are complete, the zero-mass $d = 1 + 1$ case needs both the light-ray data in order to determine the $d = 1 + 1$ theory. In the QFT the manifestation of this is the tensor factorization into the chiral degrees of freedom, which amounts to a doubling of degrees of freedom. In the next section we will see that this also leads to an exceptional behaviour in the time-like Huygens structure and the associated time-like braid group structure. So the chirally factorizing $d = 1 + 1$ situation is a guide in certain higher-dimensional aspects and stands in interesting contrast to others.

3. Central decomposition and braid group structure

In the previous section we emphasized certain analogies between the time-like algebraic structure in higher-dimensional and chiral conformal theories. For the latter, the decomposition theory has a more fundamental explanation in terms of a local plektonic superselection structure. In particular, this means that the components appearing in (10) permit a local refinement. They can be further reduced into DHR localized charge sectors of an observable local algebra which lives on S^1 and which in concrete models is generated by the energy momentum tensor, current algebras, W-algebras etc. The commutation structure of charge-carrying chiral fields obtained by the DHR method is an exchange algebra

$$F_{\alpha,\beta}(x)G_{\beta,\gamma}(y) = \sum_{\beta'} R^{(\alpha,\gamma)}_{\beta,\beta'}[c_F, c_G]G_{\alpha,\beta'}(y)F_{\beta',\gamma}(x) \qquad x > y \tag{18}$$

$$F_{a,\beta}G_{\beta,\gamma} = \sum_{\beta'} R^{(\alpha,\gamma)}_{\beta,\beta'}[c_F, c_G]G_{\alpha,\beta'}F_{\beta',\gamma} \qquad \mathrm{loc}F > \mathrm{loc}G \tag{19}$$

where in the second line we have used the more general operator formulation of AQFT and the c in $R[c_F, c_G]$ denote the dependence of the R matrices on the superselected charges c of the participating operators. The localization is always meant relative to the observables [14]. The Artin relations[5] are a consequence of the associativity of this algebra. Here the indices α, β, γ refer to projectors on irreducible DHR representation spaces. Although the latter are a refinement of the projectors appearing in the centre Z, for simplicity of presentation we maintain the same notation.

The validity of such an exchange algebra structure would supply a natural local explanation for the centre superselection rules. We will therefore postulate this structure for the time-like region in higher-dimensional conformal theories (where now $\lessgtr$ to $\mp$ time-like ordering) and, given the lack of concrete examples, test its consistency. It is reasonable to start with consistency checks in the standard Wightman framework of point-like covariant fields.

[5] In the physical literature they are often called Yang–Baxter relations, but our conceptual fidelity prohibits us from mixing the S matrix Yang–Baxter setting with the plektonic statistics concepts within the Artin braid group.

The most powerful tool of Wightman's formulation is provided by the analytic properties of correlation functions. It is well known that the complexified Lorentz group may be used to extend the tube analyticity associated with the physical positive energy–momentum spectrum. The well known BHW theorem [2] ensures that this extension remains univalued in a new complex domain and the Jost theorem characterizes its real points. Finally, space-like locality links the various permutations of the position field operators within the correlation function to one permutation (anti)symmetric analytic master function which is still univalued. The various correlation functions on the physical boundary with different operator ordering can be obtained by different temporal iε prescriptions descending to the real boundary from within the tube.

Complexifying the scale transformations, the conformal correlations can be extended into a still bigger analyticity region which even incorporates 'time-like Jost points'. But trying to find a univalued master function which links the various orders together fails in the presence of fields with anomalous dimensions and only works for observable fields which are local fields on the compactification $\bar{M}$. The latter are the analogues of chiral observables, except that, apart from (composite) free fields, one does not have algebraic examples since Virasoro– and Kac–Moody algebras do not exist in higher dimensions. The time-like braid group structure suggests that the role of the permutation group in the analytic extension from time-like points should be replaced by the braid group. The resulting ramification takes place in a region which is obtained in the analytic extension by the complex dilation group, which leaves the old univalued BHW region free of branch cuts. Therefore, the time-like braid structure is consistent with the BHW analyticity structure. Let us look at other arguments which test the consistency of the old space-like locality with the new time-like localization structure, because the problem of coexistence of these two regions is the main difference to the chiral case. This significant difference even remains if one glues together the two chiralities to a two-dimensional conformal theory. As already mentioned, it has its origin in the classical wave propagation theory, and one needs two sets of characteristic light-ray data to, for example, determine the amplitudes in a wedge region, whereas for any higher-dimensional theory (and even massive $d = 1 + 1$ propagation) one light front is enough.

A plektonic charge structure which is only visible in the time-like region would immediately explain the appearance of a nontrivial time-like centre and the spectrum of anomalous dimension. It would sort of 'kinematize' conformal interactions and reveal conformal QFTs as basically free theories if it were not for that part of interaction which sustains the time-like plektonic structure. Of course, the situation trivializes if the theory has no anomalous dimensions and nontrivial components. Analogous to [3] (see remarks at end of section 4) we conjecture that this characterizes interaction-free conformal theories which are generated by free fields[6]. What makes this issue somewhat complicated is the fact that, contrary to chiral theories, we do not have a single nontrivial example because this issue is neither approachable from the representation theory of known infinite-dimensional Lie algebras nor from the formal Euclidean functional integral method. The remaining strategy is to show structural consistency of the space-like local structure with the conjectured time-like plektonic structure and to find a new construction method (non energy–momentum tensor- or current-algebra-based, non-Lagrangian). Here we are mainly concerned with consistency arguments, and in the following we comment on how local/plektonic on-vacuum relations between two fields can be commuted through to a generic position.

[6] Note that this conjecture would be wrong in $d = 1 + 1$, since from self-dual lattice construction on current algebras one obtains models without nontrivial sectors which are different from free fields.

Assume for simplicity, as we have already tacitly done in our earlier decomposition formulae, that we are in a 'minimalistic' situation (similar to minimal models or W algebras in the chiral setting) where the field theory has no internal symmetry group. Actually, the whole discussion can be carried out in the presence of nontrivial inner symmetries, but the additional complications do not essentially alter the following consistency considerations. So we assume that the fields can be given 'time-like' charge indices $\alpha, \beta, \gamma, \ldots$ and their conjugates $\bar{\alpha}, \bar{\beta}, \bar{\gamma}, \ldots$ resulting from projectors on charge spaces, so that the decomposition is as in the chiral case where the charge projectors with the same phase factors $\mathrm{e}^{2\pi \mathrm{i}\delta}$ constitute a refinement of a central projector. Clearly, α and its conjugate $\bar{\alpha}$ contribute to the same central projector. In fact, we may take over a substantial part of the formalism and concepts of [25] if we replace the chiral translation+dilation augmented by the circular rotation generator L_0 by the spacetime symmetry group which leaves the time-like infinite point fixed (Poincaré + dilations), augmented by the generator of conformal time R_0. One would, of course, also have to think of changing the title of the old paper from 'Einstein causality and Artin braids' to 'Huygens causality and Artin braids', referring to the time-like ordering for which the conformal observables fulfil the Huygens principle of vanishing commutators. The 'on-vacuum' structure of commutation relations follows from the structure of the conformal 3-point functions

$$\left\langle H^*(x_3)G(x_2)F(x_1)\right\rangle = c_{FGH}\frac{1}{\left[-(x_{12})^2_\varepsilon\right]^{\delta_3}}\frac{1}{\left[-(x_{13})^2_\varepsilon\right]^{\delta_2}}\frac{1}{\left[-(x_{23})^2_\varepsilon\right]^{\delta_1}} \qquad (20)$$
$$\delta_1 = \tfrac{1}{2}(\delta_G+\delta_H-\delta_F) \quad \delta_2 = \tfrac{1}{2}(\delta_F+\delta_H-\delta_G) \quad \delta_3 = \tfrac{1}{2}(\delta_F+\delta_G-\delta_H)$$

where the ε-prescription was explained in the introduction. For space-like and time-like distances one concludes that (Z as defined in (10))

$$G(x_2)F(x_1)\Omega = \begin{cases} F(x_1)G(x_2)\Omega & (x_2-x_1)^2<0 \\ \mathrm{e}^{\pi\mathrm{i}(\delta_F+\delta_G)}Z^*F(x_1)G(x_2)\Omega & (x_2-x_1)^2>0 \quad (x_2-x_1)_0>0 \end{cases} \qquad (21)$$

since this relation is valid on all quasiprimary composites H. They consist, by definition, of the equal point limit of the associated primary H_{min} (lowest scale dimension operator in a superselected charge class) multiplied with a polynomial in the observable field. These composites applied to the vacuum form a dense set in the respective charge sector[7] and, hence, the on-vacuum formula is a consequence of the structure of 3-point functions. The space-like local commutativity off-vacuum is consistent with that on-vacuum, since for y time-like with respect to a space-like pair x_1, x_2 we have (here the $c(\cdot)$ denote the superselected charges as defined in (19))

$$\begin{aligned} P_\alpha F(x_1)G(x_2)H(y)\Omega &= \sum_\beta P_\alpha F(x_1)P_\beta G(x_2)H(y)\Omega \\ &= \sum_\beta P_\alpha F(x_1)P_\beta \mathrm{e}^{\mathrm{i}\pi(\delta_G+\delta_H-\delta_\beta)}H(y)G(x_2)\Omega \\ &= \sum_{\beta\beta'} R^{(\alpha\gamma)}_{\beta\beta'}(c_F,c_H)\mathrm{e}^{\mathrm{i}\pi(\delta_G+\delta_H-\delta_\beta)}P_\alpha H(y)P_{\beta'}F(x_1)P_\gamma G(x_2)\Omega \end{aligned}$$

and therefore the off-vacuum vanishing of the F–G commutator is consistent with the on-vacuum vanishing of this commutator if there holds a certain relation between $R(c_F, c_H)$ and $R(c_G, c_H)$ which is identically fulfilled for $F = G$. Similarly, one does not run into

[7] With a bit more work and lengthier formulae one can avoid the colliding point limit and use correlation functions containing three charged fields and an arbitrary number of neutral observable fields. The dependence on the observable coordinates is described by a rational function on $\bar{M}$.

inconsistencies if one tries to obtain a time-like off-vacuum F–G situation from the on-vacuum placement by commuting through a H which is space-like to the time-like F–G pair:

$$\begin{aligned} P_\alpha F(x_1)G(x_2)H(y)\Omega &= P_\alpha H(y)F(x_1)G(x_2)\Omega \\ &= \sum_\beta P_\alpha H(y)\mathrm{e}^{\mathrm{i}\pi(\delta_B+\delta_C-\delta_\beta)}G(x_2)F(x_1)\Omega \\ &= \sum_{\beta,\beta'} R_{\beta\beta'}P_\alpha G(x_2)P_{\beta'}F(x_1)H(y)\Omega \\ &= \sum_{\beta\beta'} R_{\beta\beta'}P_\alpha G(x_2)P_{\beta'}H(y)F(x_1)\Omega \end{aligned} \tag{22}$$

where in the second line we commuted F through G *before* trying to bring both to the vacuum. Since there is generally[8] no rule to commute the $P_\alpha G(x_2)P_{\beta'}$ with $P_{\beta'}H$ for $(x_2 - y)^2 < 0$, there is no way to get to the same HGF order as in the first line and hence no consistency relation is to be checked. The absence of rules for space-like commutations for projected fields protects the formalism from running into inconsistencies. If components for space-like distances were to commute then they would belong to the vacuum.

Let us also briefly look at the compatibility of the time-like plektonic structure with the conformal structure of the 4-point function of four identical Hermitian fields:

$$\begin{aligned} W(x_4,x_3,x_2,x_1) &:= \sum_\gamma \left\langle F(x_4)F(x_3)P_\gamma F(x_2)F(x_1)\right\rangle \\ &= \left[\frac{x_{42}^2x_{31}^2}{(x_{43})_\varepsilon^2(x_{32})_\varepsilon^2(x_{21})_\varepsilon^2(x_{14})_\varepsilon^2}\right]^{\delta_F}\sum_\gamma w_\gamma(u,v) \\ u &= \frac{x_{43}^2x_{21}^2}{(x_{42})_\varepsilon^2(x_{31})_\varepsilon^2} \qquad v = \frac{x_{32}^2x_{41}^2}{(x_{42})_\varepsilon^2(x_{31})_\varepsilon^2}. \end{aligned} \tag{23}$$

Whereas space-like commutation leads to functional relations for $w = \sum_\gamma w_\gamma(u,v)$ with the exchange of two fields causing a rational transformation of the u, v (apart from multiplying the w by rational u, v factors), the time-like commutation of the off-vacuum fields $x_2 \leftrightarrow x_3$ produces rational transformation together with R matrix mixing of the γ components leading to nontrivial monodromies

$$w_\gamma(u,v) = \sum_{\gamma'} R_{\gamma\gamma'}w_{\gamma'}\left(\frac{1}{u},\frac{v}{u}\right)u^{2\delta_F}.$$

As in the chiral case [25], these relations characterize equivalence classes of operators carrying the same superselected charges (conformal blocks), and the selection of individual correlation functions have to be made by using, in addition, their short-distance behaviour in terms of scaling dimensions. Despite some similarities with the chiral case, the dependence of w_γ on two instead of one cross-ratios requires the use of more elaborate techniques (Mellin–Barnes techniques, generalized hypergeometric functions) than the hypergeometric formalism (which is sufficient for the chiral one-variable cross-ratio dependence). We find it conceivable that in higher dimensions there could be deformation parameters (coupling constants) which may not show up in continuously changing anyonic phases. Here we will not pursue this matter.

Let us finally take note of what the algebraic approach can add to these consistency considerations.

[8] The exception occurs when one of the involved charges is simple (Abelian). In this case the phase factors obtained from interchanging cancel on both sides and the resulting relation is consistent.

One possible point of departure for the algebraic approach would be to start from a Doplicher–Roberts field net $\mathcal{F}$ on M and use its assumed local conformal invariance in order to construct a unique extension $\tilde{\mathcal{F}}$ on the covering $\tilde{M}$. This has been done in the work [31] where it was also shown that the extended net fulfils the important property of Haag duality which in the bosonic case is a maximalization of causality

$$\tilde{\mathcal{F}}(\mathcal{O}') = \tilde{\mathcal{F}}(\mathcal{O})' \tag{24}$$

where we have used the standard notation of AQFT: a prime on the algebra signifies the von Neumann commutant in the Hilbert space of the operator algebra, whereas on the localization region $\mathcal{O}$ (double cone, or any conformal transform thereof) it signifies the space-like complement in the global sense of $\tilde{M}$. In order to obtain information about a time-like braid group structure we need to identify an observable subnet $\mathcal{A}$ on M consisting of operators which commute with Z and which is Haag-dual for time-like distances such that the restriction of the field net to the observable net decomposes into irreducible representations of the latter which obey braid group fusion laws. The projectors onto those irreducible subspaces would then be the desired refinements of the central Z projectors by which one obtains the $F_{\alpha,\beta}$ exchange algebra operators.

A prerequisite for this idea to work in the setting of algebraic quantum field theory would be the existence of an autonomous time-like Haag-dual net which is sufficiently nonlocal in the space-like sense, i.e. sufficiently different from a space-like Haag-dual net. This is necessary in order to obtain sectors which are different from those of the space-like-based DHR theory. A theory of observables $\mathcal{A}$ on the compactification $\bar{M}$ is automatically Haag-dual, but the localization of the commutant of a double cone would consist of a space-like and time-like part [3]. We would need a dualization which involves only the time-like complement in M of double cones and which should be sufficiently nonlocal with respect to a conformal DHR dualization based on double-cone algebras defined in terms of intersections of infinitely many wedges. Algebraically, the Haag-dual net on $\bar{M}$ results from conformal transformations from the wedge algebra, whereas the double cones of the time-like dual net are obtained from intersections of the forward- with the backward-shifted light-cone algebra in M.

From a point-like field point of view, the difference in these algebras is related to the way in which smeared fields are used to generate local algebras. If they are used on M then the net at infinity is diluted because the test functions vanish on $\bar{M}\backslash M$ and in order to re-establish the duality balance one has to make the algebras in the net which do not contain points at infinity bigger. This, and the ensuing loss of space-like commutativity of these algebras, has been studied in [23]. A detailed investigation of 'cutting holes' into the circle of chiral nets and then producing new conformal nets from the Haag dualization of the punctured nets can be found in [38]. Although such studies of chiral theories cannot answer the above consistency problems in higher-dimensional theories, they do supply valuable concepts and mathematical methods for further studies.

As in many other areas of field theory, consistency problems find their satisfactory solutions only through mathematically controllable model constructions, and the structural analysis is only a preparatory step in the classification and construction of models outside the standard Lagrangian realm. Its main purpose is to prepare our intuition in an area where it is presently underdeveloped or beset by prejudices.

Thinking of what could be the right setting for such constructions, only one idea seems to offer sufficient conceptual depth and mathematical power. This is the modular localization approach based on the Tomita–Takesaki modular theory and using new physical concepts such as 'polarization-free generators' of wedge algebras. There are two ways of using that theory and both are independent of spacetime dimensions [20, 28], although special situations in low dimensions greatly favour their analytical control.

One approach is based on the fact that the S matrix is a relative modular invariant of wedge algebras and is behind the bootstrap–formfactor constructions in $d = 1 + 1$ factorizing models. Conformal theories do not have an S matrix, but there are indications that their TCP operators ($\simeq$ modular conjugations for the modular theory of the forward light-cone algebra) can be related to a simpler reference situation by generalized 'twist operators' which contain the braid information. This idea should be first tested for chiral theories since even there exists, to date, no systematic method to construct a model associated with a given plektonic superselection structure. In contrast to the standard methods which start from concrete observable algebras (Virasoro-, current- W-algebras) and introduce the charge-carrying fields as intertwiners between the irreducible representations, the modular method aims directly at the charged fields. It is analogous to the Wigner Poincaré group representation approach to free fields where the charged fields came first and the group-invariant observable algebras (which are more complicated since they involve composite fields) were later introduced only for purposes of a better structural understanding. The modular programme is in a certain sense a continuation of the Wigner approach in the presence of interactions.

4. Concluding remarks

Presently, four-dimensional conformal theories are very scarce and, in fact, do not yet exist on the same conceptual and mathematical level as chiral models. Perturbative ideas about their construction have not yet given satisfactory results. One idea which was mentioned in section 2 as not leading to physically viable conformal theories was the use of perturbative Lagrangian AdS[9] models in the AdS_5–$CQFT_4$ connection.

Recently, there have also been direct attempts to obtain nontrivial conformal theories within the family of perturbatively renormalizable supersymmetric Yang–Mills models with the vanishing Callan–Symanzik β-function (which is a necessary condition for conformal invariance in perturbation theory) [39]. There are many lowest-order perturbative calculations for gauge-invariant quantities (often involving additional approximations), but they have not yet reached a level where they could be used for a test with the braiding ideas in this paper. Even the conjectures on the existence of certain 'protected' quantities are not very clear. Protected objects, by definition, do not receive any contributions from the interaction, i.e. they retain their zero-order free-field values. A chiral illustration of a protected quantity would be a nontrivial chiral theory which has an energy–momentum tensor with the free-field value $c = 1$, of which there exist plenty. 'Nontrivial' here simply means that there are other unprotected quantities in the model. What is conceivable is a protection of an entire subalgebra, as in this analogy. A protection of only certain correlation functions on the other hand (say the two- or three-point normalization constant of a certain operator) which does not follow a (supersymmetric) charge rule which characterizes a subalgebra of operators is hardly reconcilable with one's understanding of the omnipresence vacuum polarization in interacting QFT. So the question of the quality of arguments about protection remains open.

The crucial question for a future comparison with the time-like braid group structure is whether the perturbation theory can be pushed far enough so that one can extract anomalous dimensions. If it works, it is probably limited to models with an Abelian braid group phase, which in the analogy to chiral models would mean something analogous to the massless Thirring model but not to, for example, a minimal model without a coupling deformation parameter.

[9] Necessarily with point-like fields, because there is no known Lagrangian field theory for extended 'fields'.

Since it is not unrealistic to expect that the first conceptually and mathematically controllable four-dimensional nontrivial models will be conformal (because they stay close to free-field theories without being identical to them), this line of research has a certain urgency and importance for the future of QFT.

Another fascinating, but at the same time very speculative, idea which emerges from the present setting on the structure of higher-dimensional conformal theories is the suggestion that there may be charge superselection rules and inner symmetries which do not respect the inner/spacetime factorization pattern of the Coleman–Mandula theorem and (*a fortiori* not its prerequisites) even after adjusting the prerequisites in order to incorporate supersymmetry. Braided charges and their fusions certainly have a very different structure than multiplets of fields on which compact groups act. In fact, a time-like braided structure would inexorably remain linked with spacetime and dynamics. We have become so used to thinking in terms of non-Abelian symmetry groups that it is helpful to recall from time to time that there actually does not exist a single exact non-Abelian continuous flavour symmetry in nature. So to look for explanations of the observed regularities outside group theory may not be as absurd as it appears at first sight. But the idea that the observed regularities may be remnants of time-like braidings remains a farfetched and wild speculation as long as there is no understanding of how conformal theories can be naturally related to particle models in a more controllable way than assigning to them a short-distance universality class.

Note added in proof. Meanwhile Rehren has extracted a 'mixed statistics group' from the consistency relations of the present paper. This group combines the time-like braid group with the space-like permutation group (see also [40]).

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